The Importance and Value of Real-Time Simulation

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ABSTRACT: Two developments are influencing design engineers in the use of simulation, especially, real-time simulation. Recent advances in hardware speed make it possible to simulate in “real time” complex systems that previously could not be simulated in real-time. The advent of convenient-to-use programming languages like Simulink and SystemBuild has allowed a new class of engineers the ability to develop complex simulations with less knowledge of the underlying physics than was previously possible.

Several new developments in real-time simulation will be discussed and examples given. On-line computation of errors (while the simulation is running) will be discussed. These errors are primarily due to truncation errors of the mathematical integration formulas and until now could be estimated for all digital simulations but was nearly impossible to estimate for a simulation with hardware-in-the-loop (HIL). A new method makes on-line measurement of errors (not an estimate) possible for an HIL simulation.

Until now, HIL simulations were required to be deterministic with fixed time-step sizes. Deterministic meaning the simulation is synchronous in time with the time to compute a time step fixed and known. A new technique of asynchronous simulation that uses time-tagged variables and allows a real-time HIL simulation to have variable time steps will be discussed.

1. Introduction

This paper will review and summarize two recent developments in real-time simulation that could improve both accuracy and speed of simulations that use hardware-in-the-loop (HIL) simulations.

2. On-line Measurement of Dynamic Errors in Real-time Simulations

In the real-time simulation of dynamic systems, including hardware-in-the-loop (HIL), the direct determination of simulation errors resulting from finite numerical integration step sizes and input/output frame rates has always been elusive. The method for dynamic error measurement as described in [1] is based on comparing two real-time simulations, one with the nominal integration step size and the second with an augmented, or slightly different, step size. Accurate interpolation formulas [2] are then used to convert the discrete times associated with the data sequence outputs from the second simulation to the discrete times for the first simulation.

This permits a direct computation of the difference between the two solutions at common discrete times, from which the deviation of either simulation from the solution at zero step size can be calculated with a simple formula. Although only approximate, this procedure results in a calculated dynamic error that is surprisingly accurate as is shown in example simulations in [1]. Furthermore it permits direct determination of the dynamic errors associated with each frame rate, which, in turn, permits the determination of an optimal frame-rate ratio. In a multi-processor simulation, it allows the dynamic errors associated with each processor simulation to be determined separately. It also permits direct determination of the effect of input-output sample rates on the overall dynamic accuracy.

With the use of extrapolation methods [2] already developed for asynchronous simulation, all of this can be achieved on-line in a real-time simulation environment.

3. On-line Error Measurement

Reference [1] describes a second-order flight control system where two simulations are performed at two different time steps, $h=0.0045$ and $h=0.005$. Figure 1 shows the estimate of the error in $\delta_c$, control surface actuator output, for the two different step sizes. This error can be calculated by a relatively simple formula and could be computed on-line with relatively low overhead.

(Figure 1)
Figure 2 shows the error in \( \delta_e \) from an on-line calculation plotted on top of the same error using a reference non-real time RK-4 simulation. The agreement between a relatively simple on-line calculation formula and an RK-4 reference is remarkable considering that the on-line equation is an approximation.

4. An Asynchronous Simulation Methodology

In the real-time simulation of dynamic systems it has been traditional to use a fixed time step for numerical integration. The overriding reason for not using a variable step size is the possibility that the mathematical step size can become smaller than the computer execution time for the calculations involved in a given integration frame. This in turn means that the simulation output at the end of that step will fall behind real time.

Another reason for choosing a fixed time step in real-time simulation is the compatibility with fixed sample rates when dealing with real-time inputs and outputs. On the other hand, when conditional statements are present in the program that is executed every integration step, the frame execution time will not be constant. Also the refresh operation associated with dynamic random-access memory will, in general, cause small variations in the execution time for integration frame, as will the utilization of cache memory, depending on the frequency of cache “hits.”

The mathematical step size in a fixed-step real-time simulation must of necessity be set equal to the maximum expected value of the frame execution time, which often may not even be known in a complex simulation, in order to ensure the availability of simulation outputs in real time.

The argument given above for simulations using fixed step integration based on the fixed sample–rate requirements for real-time inputs and outputs can be waived if we are willing to use the extrapolation methods described in reference [2]. In particular, the extrapolation formulas permit both real-time inputs and outputs to be corrected for any lack of synchronization with the computer-simulation frame times.

This in turn allows variable-step integration methods to be employed for improved computational efficiency. It also permits the real-time computer simulation to be run with the mathematical step size for each successive integration step set equal to the measured execution time of the previous step. With this procedure, the simulation is always able to keep up with real time, at least to within a fraction of the integration step size.

The procedure also permits the real-time integration step size to be set automatically by the software, without user intervention. On-line error measurements, as described above, can be employed to alert the user if the simulation errors ever become excessively large.

5. An Example Asynchronous Simulation

Consider a second order system simulation using the widely used integration formula second-order Adams-Bashforth (AB-2) predictor method [3, 4]. A variable time step is used where the time step used at each integration step is as shown in Figure 3. The time step used in the AB-2 integration formula varies between 0.16 and 0.24. This could be representative of varying execution time in a simulation due to conditional statements or other factors in the simulation program.

Figure 3 shows the simulation response with the variable time steps compared to the exact solution. The variation in step size is evident in the uneven time-axis spacing of the data points in Figure 4. The accuracy of the simulation is easier to see in Figure 5 where the simulation error is plotted directly, along with the error using a fixed step size of \( h = 0.2 \) (the mean value of the variable step size) and \( h = 0.24 \) (the maximum value of the variable step size).
Note that the errors for the variable-step case are essentially the same as those for the fixed step when that step is equal to the mean, $h = 0.20$. On the other hand, if the fixed step is set to accommodate the maximum variable step size, the errors are significantly larger, i.e., by the ratio $(0.24/0.2)^2$.

6. Summary  
An overview has been given of on-line calculation of errors in a dynamic simulation that uses an error calculation formula with little computational overhead and is surprisingly accurate. Additionally, an overview was given of a new asynchronous simulation methodology that has advantages of using a variable time step in a real-time simulation.

7. References

8. Biographies
Dr. Maurice Snyder. Dr. Snyder is Director of Asia Operations, Applied Dynamics International. His background includes a PhD in Electrical Engineering and extensive experience in international marketing and real-time simulation systems.