Enhancing Empirical Ballistics Modelling Techniques To Provide Solutions For Naval Gunfire Support

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Abstract. Naval Gunfire Support requires the interaction between a ship gun and reports from a ground spotter to provide a firing solution. The firechain analysis for the NGS operations include a ballistics model representing the actions of the gun, a model to represent the spotter on the ground making manual observations and an approximation to represent the accuracy of the gun aim and the accuracy of the shot. This paper presents how a scalable fidelity model can be built from first principles.

The projectile trajectory is represented through the use of external ballistics and Newton’s Laws of Motion equations. Additional to the projectile modelling there is a requirement for reporting on the accuracy of the shot. In typical engagements it is reasonable to expect that shot accuracy will improve as subsequent bursts are fired. This is due to feedback of information to the gun operators regarding the fly-outs of previous rounds, allowing compensation for system errors. Sources of error include alignment bias, tracking errors, aimpoint prediction errors, pointing errors, and ballistic variation, which together determine the round-to-round dispersion and aiming error. In particular, the improvement in accuracy is modelled as a reduction in the probabilistic aimpoint error with each subsequent burst. Additional fidelity to the model is achieved through the use of a user-defined burst size for each round of engagement.

Analysis with a stochastic Monte Carlo approach can lead the user into understanding the predicted capability of a naval gun against a land-based target. This allows the operational aspects of Naval Gunfire Support to be addressed, with proposed methodologies, including cumulative probability charts, to determine mission success in terms of accuracy of shot, time to achieve mission objective and overall mission assessment.

1. INTRODUCTION (200 WORDS)

As a warfare area, Naval Gunfire Support (NGS) involves the support of land forces through the bombardment of land positions with a naval gun. An NGS engagement requires the interaction between a ship gun system and an observer who provides target position information, spots the fall of rounds to provide targeting correction and carries out damage assessment (see Cramp (2006) for an extended discussion).

Modelling of the engagement can provide insights into the operational impacts on the capability of a naval gun system to participate in Naval Gunfire Support. Of particular interest is analysis of temporal changes on the firechain. The total time taken from the moment an observer provides initial target coordinates until the first rounds land on target is the sum of many timing intervals, including the time for computing a firing solution, time to slew and fire the gun, projectile time of flight (TOF) and time to assess the accuracy of shot and report targeting correction information. This paper discusses a model (hereafter referred to as The NGS Model) constructed to simulate these timing chains, as well as provide approximations for the accuracy and outcome of each shot, in order to characterise the NGS capability of a naval gun system.

The paper is organised as follows: Section 2 describes the flow of operations in The NGS Model, including a discussion of an external ballistics model used to generate ballistic flight paths of gun rounds; Section 3 describes the application of errors for generating approximations of shot accuracy and outcome; and Section 4 illustrates the metrics that are output from the model to provide measures of effectiveness for describing NGS capability.

2. DESCRIPTION OF MODEL

The NGS Model has been designed to allow flexibility in the operational characteristics of firing of a naval gun and uses a simple input–execution–output flow. The user populates parameters to represent both physical and functional components of the gun system, thereby characterising the accuracy and timing performance during the engagement. Execution of the model is then automatic, carrying out all aspects of the engagement and ceasing once the target is neutralised or the user-defined maximum number of bursts has been fired. The outcome of the engagement in terms of time and number of bursts to achieve a hit and accuracy of each round are reported.

The NGS Model is a discrete event simulation and calls on a collection of component models to simulate an NGS engagement from the time the first target designation is reported to the gun operators. The flow of operations carried out by The NGS Model is illustrated in Figure 1. The engagement process for each burst includes computing a firing solution from a pre-defined firing table, slewing and firing of the gun, ballistic round fly-out, determination of fall of shot and spotter kill assessment and correction reporting.
2.1 The NGS Model

The engagement begins at time zero with the reported position of a target. The target is either a stationary or moving land target. If moving, the target is limited to straight line trajectory segments defined by heading and speed. A user-defined range and bearing error is applied to the actual target position at time zero to generate the initial reported range and bearing to the target. The gun immediately slews to an approximate azimuth and elevation required for firing with the time taken determined by user-defined slew rates.

Following the initial slew the target range and bearing is updated from the specified trajectory, with appropriate errors applied. The approximate time of flight (TOF) for a round to fly out to this location is called from the firing table and used to schedule the arrival of each round on target. The user specifies a ‘buffer-time’ to be applied in this scheduling, which compensates for the time taken to adjust the azimuth and elevation of the gun between firings to track moving targets.

If a moving target is modelled the projected intercept point (PIP) for the target is extrapolated given the reported speed and heading of the target at the last update. A refined estimate of the round TOF is called from the firing table and the gun then adjusts azimuth and elevation to fire to this PIP. A hold-off time is imposed (determined by the buffer-time, the accuracy of the initial TOF estimate and the time to adjust the gun position) and the gun fires the first burst of rounds at the user-defined firing rate.

To model the inherent inaccuracies in a gun system the user specifies azimuth and elevation gun pointing errors (in microradians) and a charge variation error (a percentage). These errors are applied to determine the actual gun azimuth, elevation and muzzle velocity for each shot. Over a large number of firings the gun pointing azimuth and elevation and the muzzle velocity will be normally distributed with standard deviation equal to the user-defined errors.

The actual gun elevation angle and muzzle velocity are passed to the ballistics model to determine the fly-out of the round, as described in Section 2.2. The round range and time-of-flight are returned and used along with the actual gun azimuth to determine the fall-of-shot for each round. Figure 2 shows the outcome from an engagement of a stationary target. In this case, four bursts were fired, each of four rounds.

To determine if a round terminates in a hit or a miss the closest point of approach (CPA) to the target is calculated by updating the target position at the time the round impacts the ground. The CPA is used in conjunction with the user-defined target radius and a target vulnerability model, described in Section 3, to determine the outcome.

Figure 1: Flowchart of the operations carried out by The NGS Model

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The Naval Gunfire Support (NGS) Model

- Target position reported
- Gun slews to approximate Az/El
- Reported target position updated
- TOF to target current position called from firing table.
- Schedule intercept time (Rnd load time + buffer time + TOF)
- Round fire time computed (Scheduled intercept time – TOF)
- Gun slews to required Az/El – called from firing table
- Fire round
- Round CPA to target calculated
- Vulnerability model called to decide if kill achieved
- Correction model applied to reduce reported range/bearing errors for next burst
- Hold-off time applied
- Firing solution update time applied
- Slewing time applied - (moving targets only)
- Slewing time applied
- Round load and fire time applied
- Round flight time applied
- Time delay for observer to spot fall of shot and perform kill assessment applied
- Target killed?
  - No
  - Yes
  - Report burst number and time at which kill is achieved

The NGS Model

- Is target moving?
  - No
  - Yes
  - Projected intercept point (PIP) computed
  - Refined TOF to the PIP called

For bursts of rounds

- Target designation process time applied
- Slewing time applied
- Round CPA to target calculated
- Vulnerability model called to decide if kill achieved
- Time delay for observer to spot fall of shot and perform kill assessment applied
- Report burst number and time at which kill is achieved

- Correction model applied to reduce reported range/bearing errors for next burst
- Hold-off time applied
- Firing solution update time applied
- Slewing time applied
- Round load and fire time applied
- Round flight time applied
- Time delay for observer to spot fall of shot and perform kill assessment applied
- Report burst number and time at which kill is achieved
Figure 2: Outcome of single trial of The NGS Model; the red markers indicate the fall of shot for each gun round.

Following the fire and fly-out of the first burst of rounds, an assessment delay is imposed to model the time taken for a spotter to perform kill assessment and report fall-of-shot errors back to the gun operators. This is modelled by an offset Rayleigh distribution with user-defined minimum time and standard deviation. If no hit occurred the firing solution recalculates and bursts are fired until a hit is achieved or the maximum number of bursts is reached. Spotter correction may optionally be applied to reduce the reported range and bearing errors for subsequent bursts, described in Section 3.

2.2 The Ballistics Model

A key component in the operation of The NGS Model is the fly-out model for ballistic projectiles. This is an external component built up from a first principles consideration of the kinematics of projectiles in flight, but with an emphasis on achieving a suitable balance between fidelity and computational efficiency. External ballistics analysis is often carried out with a full six degrees of freedom (6DOF) approach, resulting in very high fidelity, but with corresponding demands on developer expertise and computing time. The requirements of this modelling effort were for greater fidelity in the time components of the engagement sequence, and lower fidelity in the endgame outcome of each shot. To this end, the ballistics model was developed to provide realistic firing ranges and round time-of-flights for variable gun elevation angles, thus improving fidelity in gun slewing time and round TOF for the overall model.

The ballistics model is a time-stepping simulation and computes the motion of the round as determined by the effects of two forces, gravity and drag. The model assumes a five inch naval gun round of 30 kg and a homogenous environment consisting of dry air with a constant temperature lapse rate of 0.0065 K/m, sea-level air temperature of 288 K, sea level air pressure of 101325 Pa and zero wind. The inputs to the model are the gun elevation angle and the muzzle velocity, supplied by The NGS Model. The magnitudes of the forces acting on the projectile in each time-step are determined by the initial conditions of the projectile at the start of the step: velocity, flight angle, height, range; and the environmental conditions throughout the step: gravitational acceleration and air density.

For the first time step, the initial velocity is the muzzle velocity, initial flight angle is the gun elevation angle, initial height is the round release height (given the gun mount height and barrel length) and initial range is zero. The gravitational acceleration and air density are calculated for the given projectile height from standard physical equations. The drag force is then given by The Drag Equation (Serway & Beichner, 2000):

$$F_d = -\frac{1}{2} \rho v^2 A C_d$$

where:  
$\rho$ = air density  
$v$ = projectile speed  
$A$ = projectile cross-sectional area  
$C_d$ = drag coefficient

The drag coefficient is dependent on the geometry of the projectile and itself varies with speed. In the field of external ballistics many generic drag models have been developed from empirical evidence to characterise projectiles of a particular shape (Schaefer, 2006). For artillery rounds, the model commonly used is the G2 Drag Model. This corresponds to a projectile with shape depicted in Figure 3 below, and matches well with the shape of rounds fired from the naval gun considered.

![G2 Projectile](image)

Figure 3: Comparison of geometries of a typical naval gun round and the G2 projectile

The variation of the drag coefficient as a function of speed for the G2 model is shown in Figure 4 (blue data points). Note the erratic behaviour in the region around 300 m/s as the projectile transitions from supersonic to subsonic speeds. Conveniently in the case of the naval gun considered, the projectile velocity does not drop below about 260 m/s for elevation angles between -5° and 47°, and muzzle velocities above 800 m/s, and so it is possible to approximate the data set with a log-
normal function (depicted by solid red line) (Weisstein, 2008, 1).

Therefore, the aimpoint error is dictated by the reported range and bearing errors and, for moving targets, errors caused by extrapolating the target trajectory. It is assumed that the aimpoint error can be reduced for subsequent firings of the gun by feedback from an observer spotting the fall of shot and providing correction. This is modelled by a reduction in the reported range and bearing errors for each burst fired. The correction function is:

\[ \sigma_n = \frac{\sigma_1}{s\sqrt{n}} \]

where: \( \sigma_n \) = the reported range/bearing error for the \( n \)th burst. \( (n > 1) \)
\( \sigma_1 \) = the initial reported range/bearing error.
\( s \) = the correction factor.
\( n \) = the burst number.
(Note that the user-defined reported range/bearing error is used for the first burst.)

The effect of this function is illustrated in Figure 7 below. For the parameters as listed, the reported range and bearing errors for 12 subsequent bursts is shown. It can be seen that the function causes a rapid decrease in the error for early bursts, and smaller decreases later.

If correction modelling is applied, the user inputs the correction factor to determine the degree of improvement desired. The impact of the correction factor parameter is illustrated in the sample output in Section 4.

The radial dispersion of a burst is defined as the standard deviation of the delta between the PIP and the fall of shot for all rounds in a burst. That is, the radial dispersion is the radius of a circle centered on the PIP (aimpoint) within which approximately 68% of the rounds in the burst landed. In the model, the radial dispersion is determined by the azimuth and elevation gun pointing errors and the charge variation error. These three parameters are used to encompass the random errors introduced by all variable impacts on a naval gun system, such as environmental effects, barrel wear, charge variation, alignment and stabilisation errors.
In Section 2.1 it was stated that a target vulnerability model is applied to determine the outcome of each round. This model allows the user to address the lethality of rounds landing nearby to ‘soft’ or ‘hard’ targets. The vulnerability as a function of distance from the target is modelled by a logistics equation (Weisstein, 2008, 2):

\[ P(\text{Kill}) = \frac{1}{1 + \left( \frac{1}{P_0} - 1 \right) e^{-rD}} \]

where:  
- \( P_0 \) = the \( P(\text{Kill}) \) given a direct hit occurs  
- \( r \) = the roll-off rate  
- \( D \) = the distance from the edge of the target

If the CPA of the round is less than the target radius, a direct hit occurs, and the probability a kill is achieved is \( P_0 \). If the CPA is greater than the target radius the roll-off rate, \( r \), dictates how likely the target is to survive nearby rounds. If \( r \) is large a direct hit is required to neutralise the target. If \( r \) is small, rounds even many metres away can prove lethal. Figure 8 shows the vulnerability model for \( P_0 = 1.0 \) and \( r = 1.5 \).

4. MODEL OUTPUTS

A single execution of The NGS Model carries out an engagement given the parameters defined by the user. The outputs from the model are the result of each round fired (hit/miss), the aimpoint error and radial dispersion for each burst, the burst number on which the first hit occurred, and the total time taken to achieve the first hit. However, this output is a single statistical trial resulting from the random draws against the multiple probabilistic variables in the model. By executing a large number of independent trials the long run average performance is determined. This Monte Carlo analysis is used to produce probability metrics to characterise the gun system performance.

The primary output from a Monte Carlo run of The NGS Model is the cumulative probability of a hit on target as a function of burst number or time. This metric is used as a measure of effectiveness in modelling the capability of the gun system.

The sample output figures below are produced from a Monte Carlo run of 100 trials; Figure 9 is a histogram of the burst number on which the first hit is achieved and Figure 10 shows the cumulative probability of achieving a hit (P(kill)) vs time.

The model can also be tailored to execute a batch of Monte Carlo runs to perform parametric analyses. In
Section 3 the correction function was described; the impact of the correction factor on the outcome of an engagement is illustrated in the output in Figure 11 below. A Monte Carlo run of 200 trials of the model was executed for each of the values of the correction factor from 0.75 to 1.50, in addition to no correction applied.

![Cumulative Pk vs Burst number](image)

**Figure 11**: Parametric analysis of effect of varying the correction factor on the cumulative probability of a hit on target vs burst number

It can be seen that the cumulative P(kill) rises more rapidly for higher values of the correction factor and therefore the correction factor controls the rate at which the gun operators ‘zero-in’ on the target. It can also be seen that when no correction is applied the P(kill) increases at a more steady rate, and that even after ten bursts, the P(kill) has only reached 80%. This is to be expected given that the size of the aimpoint error is not systematically reduced as more bursts are fired.

5. CONCLUSION

The NGS Model has been constructed to meet the particular needs of modelling operational impacts on Naval Gunfire Support engagements. Of greatest importance in this effort is accuracy in time components of the engagement sequence. To this end, greater fidelity has been assigned to time-critical aspects of the engagement, such as gun slewing times, round times of flight, and spotter kill assessment and reporting delays, rather than precise modelling of environmental effects and projectile kinematics. The model is thus considered to be fit for purpose in that it offers sufficient fidelity and flexibility to undertake the desired analysis of operational impacts and provide appropriate metrics describing the performance of the gun system.

An abbreviated description of the model components has been provided by following the sequence of operations as executed by the code, with the primary mathematical models underpinning model events highlighted. In general, the application of these mathematical functions includes a stochastic element, giving rise to statistical variations as experienced in actual NGS engagements. It was demonstrated that a Monte Carlo analysis using this model enables key measures of effectiveness, including cumulative probability of a hit on target, to be generated for the purpose of assessing NGS capability.

REFERENCES


